

Hidden Variables and the Large-Scale Structure of Spacetime

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We discuss how to embed quantum nonlocality in an approximately classical spacetime background, a question which must be answered irrespective of any underlying microscopic theory of spacetime. We argue that, in deterministic hidden-variables theories, the choice of spacetime kinematics should be dictated by the properties of generic non-equilibrium states, which allow nonlocal signalling. Such signalling provides an operational definition of absolute simultaneity, which may naturally be associated with a preferred foliation of classical spacetime. The argument applies to any deterministic hidden-variables theory, and to both flat and curved spacetime backgrounds. We include some critical discussion of Einstein's 1905 'operational' approach to relativity, and compare it with that of Poincaré.

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1 Introduction

‘The simultaneity of two events, or the order of their succession, the equality of two durations, are to be so defined that the enunciation of the natural laws may be as simple as possible.’

– Henri Poincaré (1905a).

‘What really matters is not merely the greatest possible simplicity of the geometry alone, but rather the greatest possible simplicity of all of physics (inclusive of geometry).’

– Albert Einstein (1949a).

This article concerns the structure of spacetime on large scales, in the context of hidden-variables interpretations of quantum theory. In particular, we shall be addressing the question of how macroscopic quantum nonlocality can be embedded in an approximately classical spacetime background. We argue that this question must have an answer, regardless of what the underlying microscopic theory of spacetime may turn out to be, and further, that the most natural answer is to introduce an absolute simultaneity, associated with a preferred foliation of classical spacetime (flat and curved).

The introduction of an absolute simultaneity, to accommodate the nonlocality of quantum theory over macroscopic distances, was suggested in particular by Popper (1982), Bohm and Hiley (1984), and Bell (1986, 1987). This proposal is often regarded as unsatisfactory, because quantum nonlocality cannot in fact be used for practical signalling at a distance, making the preferred rest frame undetectable in practice. As Bell put it (1986: 50):

‘It is as if there is some kind of conspiracy, that something is going on behind the scenes which is not allowed to appear on the scenes. And I agree that that’s extremely uncomfortable’ (Bell 1986).

However, Bell missed the point that, in (deterministic) hidden-variables theories, the inability to use quantum nonlocality for remote signalling is not a fundamental constraint. It is, rather, a peculiarity of a special ‘quantum equilibrium’ distribution of hidden variables. For more general, ‘non-equilibrium’ distributions, practical nonlocal signalling is indeed possible (Valentini 1991a,b, 1992, 1996, 2001, 2002a,b,c). From this perspective, our inability to detect the preferred rest frame is *not* an uncomfortable conspiracy seemingly built into the laws of physics; it is simply an accident of our living in a state of quantum equilibrium, whose statistical noise masks the underlying nonlocality.

In our view, if one wishes to appraise the structure of spacetime at a more fundamental level, then this should be done taking into account the wider, explicitly nonlocal physics of quantum non-equilibrium, rather than merely in terms of the statistical predictions of quantum theory, which are not fundamental but merely contingent on a special distribution of hidden variables.

We shall argue that non-equilibrium instantaneous signalling defines an absolute simultaneity, within the approximately classical spacetime defined by

macroscopic rods and clocks, and that fundamental local Lorentz invariance should be abandoned. It will also be suggested that the widespread excessive reluctance to consider abandoning (local) Minkowski spacetime has its origin in the unfortunate ‘operational’ approach to relativity taken by Einstein in 1905.

1.1 Status of Lorentz Invariance in Contemporary Physics

Locally speaking, the relativity of simultaneity is usually regarded as a consequence of (local) Lorentz invariance. Before considering quantum nonlocality, then, let us first briefly review the current status of Lorentz invariance in other areas of physics.

In high-energy physics, the status of Lorentz invariance is certainly open to question. The divergences of quantum field theory can be most straightforwardly eliminated by introducing a short-distance cutoff, which breaks Lorentz invariance. This suggests that it would be an advantage if Lorentz invariance were not fundamental. It is also sometimes argued that exact Lorentz invariance is experimentally inaccessible because the boost parameter (for the non-compact Lorentz group) has an infinite range which can never be probed uniformly (Jacobson and Mattingly 2001).

However, like renormalisability, Lorentz invariance did play a key historical role in the development of the standard model of particle physics. Yet, the fact that only renormalisable terms appear in the Lagrangian of the standard model is now generally regarded as merely an accident of the low-energy limit, where non-renormalisable terms are screened off in the infra-red (Weinberg 1995: 519). Clearly, the mere fact that a property played a crucial historical role in constructing our current theories is not a conclusive argument for that property to be fundamental.

Possibly, in high-energy physics, Lorentz invariance will eventually acquire a similar status to that of renormalisability, as a mere low-energy symmetry (Nielsen and Ninomiya 1978; Chadha and Nielsen 1983; Allen 1997; Moffat 2003). In any case, non-Lorentz-invariant extensions of the standard model have been considered in detail (Colladay and Kostelecký 1998; Coleman and Glashow 1999), where terms in the Lagrangian breaking Lorentz symmetry might come from deeper physics beyond the standard model. A number of experiments searching for such effects have been performed, while further experiments are underway or being planned (for reviews, see Kostelecký 2002).

Further questioning of Lorentz invariance comes from quantum gravity, in which the possibility of a minimum length at the Planck scale suggests that Lorentz invariance might emerge only as an approximation on larger scales (Kostelecký 2002). Indeed, it has been suggested that peculiarities in cosmic-ray data, together with other astrophysical anomalies, might be a sign of a breakdown of standard special-relativistic kinematics, possibly due to quantum gravity effects (Amelino-Camelia 2002). In addition, models of classical gravitation with a ‘dynamical preferred frame’ have been considered (Jacobson and Mattingly 2001).

One could certainly question the above motivations for considering a breakdown of local Lorentz invariance. Still, it is clear that Lorentz invariance is far from being a dogma in the context of high-energy physics or quantum gravity. Rather, it is often regarded as one important symmetry among others, whose status (approximate or fundamental) is a matter for experiment. And in comparing experiments with theory, it is helpful to have models incorporating violations of Lorentz invariance (as well as models incorporating violations of other important symmetries such as CPT invariance; Mavromatos 2004).

1.2 Quantum Nonlocality

In contrast, in the context of quantum foundations, attachment to Lorentz invariance tends to be more dogmatic. A number of authors insist that a realistic quantum physics should be ‘seriously Lorentz invariant’, in the sense that Lorentz invariance should be fundamental, and not merely phenomenological or emerging in some limit. This contrast is remarkable, because it is precisely in quantum foundations that there is arguably the strongest motivation of all for abandoning fundamental Lorentz invariance: the experimental detection of quantum nonlocality, through the observed violations of Bell’s inequalities.

As emphasised by Bell (1987), quantum theory is incompatible with locality, *independently* of any assumption about the existence of hidden variables. Given a pair of spin-1/2 particles in the singlet state, a quantum measurement of z -spin at one wing B allows the experimenter at B to predict in advance the outcome of a quantum measurement of z -spin at the distant wing A (in ideal conditions). As was first argued by Einstein, Podolsky and Rosen (1935) (using a somewhat different example), if locality is assumed, then changing what is done at B (from a z -spin measurement to some other measurement) cannot affect the outcome at A , and therefore the z -spin outcome at A must be determined in advance regardless of what measurement is performed at B . Having reached the conclusion that the outcomes at A and B are locally determined, one can then run a Bell-type argument, showing that their statistical correlation is incompatible with the predicted (and observed) quantum correlation. If we leave aside the many-worlds interpretation,¹ it follows that locality is incompatible with quantum theory. Note that in this argument, determinism at each wing is not assumed, but *deduced* from the assumption of locality (Bell 1987: 143).

There is then strong evidence (again, if we leave aside the possibility of many-worlds) that in the above set-up the physical processes at A and B are not independent, no matter how remote A and B may be from each other. This raises the question of how such nonlocally-connected processes may be embedded into the structure of standard relativistic spacetime.

It is sometimes suggested that, instead of accepting the existence of superluminal influences, the whole issue could be avoided by assuming that our

¹The Bell inequalities do not apply in the many-worlds theory, because their derivation assumes that a quantum measurement has only one outcome.

classical spacetime is merely emergent. Now, it may well be true that classical spacetime is emergent (for example from a deeper discrete structure). However, this does not affect the issue at all. The EPR-Bell correlations observed in the laboratory take place at macroscopic distances (for example 12 m; Aspect *et al.* 1982), involving photons with quite ordinary energies (for example visible photons of wavelength ~ 500 nm; Aspect *et al.* 1982). The detection events are recorded as taking place in a region of space and time whose structure may be operationally defined by macroscopic rods and clocks, in the laboratory where the experiment is performed. There is no doubt that the structure of spacetime in that laboratory, as defined by macroscopic rods and clocks, is to very high accuracy well-described by standard relativity theory. One may then ask, *in the approximation* where the background spacetime is approximately classical, how the events or outcomes recorded at A and B are to be embedded in the background spacetime. Whatever the final theory underlying spacetime (if there is one) turns out to be, this question must have an answer, and the aim of this article is to provide one.

1.3 Non-Equilibrium Hidden Variables

We shall consider the issue from the standpoint of deterministic hidden-variables theories. These provide a mapping from initial hidden parameters λ to final outcomes of quantum measurements. The mapping depends on the macroscopic settings defining the experimental set-up. For entangled quantum states, the mapping is nonlocal, in the sense that outcomes at one wing depend on experimental settings at the distant wing (in at least one direction; Bell 1964). Thus, the nonlocality is clearly present in the underlying dynamics associated with the mapping. Instantaneous signalling is not possible in such theories, however, *provided* the initial hidden parameters λ have a special ‘quantum equilibrium’ distribution $\rho_{QT}(\lambda)$. This distribution is chosen so that the resulting statistics of quantum measurement outcomes agree with quantum theory.

As we shall discuss in section 3, once one is given a deterministic hidden-variables theory for individual systems – where mathematically the theory is defined by the mapping from λ to outcomes – then there is no conceptual reason why one should not consider the physics of more general ‘non-equilibrium’ distributions $\rho(\lambda) \neq \rho_{QT}(\lambda)$. For such distributions, nonlocality is present not only for individual outcomes, but also at the statistical level: the marginal statistics at one wing of an entangled state do (generically) depend on measurement settings at the distant wing (in at least one direction). In such circumstances, with $\rho(\lambda) \neq \rho_{QT}(\lambda)$, practical nonlocal signalling would be possible (Valentini 1991a,b, 1992, 1996, 2001, 2002a,b,c).

If one takes deterministic hidden-variables theories seriously, then one is driven to conclude that our inability to send instantaneous signals is merely an accident of our living in a time and place where the parameters λ have the special distribution $\rho_{QT}(\lambda)$, for which statistical noise happens to erase (on average) the effects of nonlocality. This state is roughly analogous to a state of global thermal equilibrium in classical physics, in which it would be impossible to convert heat

into work (as this requires differences of temperature). In such a world – in a state of thermodynamic ‘heat death’ – the inability to convert heat into work is not a law of physics, but rather a contingent feature of the state of thermal equilibrium. Similarly, in our view, the absence of superluminal signalling in our world is not a law of physics, but rather a contingent feature of the state of quantum equilibrium (Valentini 1991a,b, 1992, 1996, 2001, 2002a,b,c).

Non-equilibrium deviations $\rho(\lambda) \neq \rho_{QT}(\lambda)$ might have existed in the very early universe, with the relaxation $\rho(\lambda) \rightarrow \rho_{QT}(\lambda)$ taking place during the violence of the big bang (Valentini 1991a,b, 1992, 1996, 2001, 2002a,b,c; Valentini and Westman 2005). In effect, a hidden-variables analogue of Boltzmann’s ‘heat death’ may have actually taken place in our observable universe. However, relic cosmological particles that decoupled at sufficiently early times might still be out of equilibrium today (Valentini 1996, 2001). It has also been suggested that quantum non-equilibrium might be generated in systems that are entangled with degrees of freedom located behind the event horizon of a black hole (Valentini 2004a,b).

In any case it is certainly true that, from a hidden-variables perspective, quantum theory is merely the phenomenological description of the statistics of a special state with $\rho(\lambda) = \rho_{QT}(\lambda)$. In principle, there exists a wider (and explicitly nonlocal) physics of non-equilibrium with $\rho(\lambda) \neq \rho_{QT}(\lambda)$.

2 Physical Structure of Spacetime

The structure of spacetime at the most fundamental level should be defined in terms of the physics at the most fundamental level. In a deterministic hidden-variables theory, emergent properties of the quantum equilibrium state (such as locality) have no fundamental status. The truly fundamental and nonlocal physics is visible only in non-equilibrium. Therefore, a fundamental appraisal of spacetime structure must be in terms of non-equilibrium physics, taking into account instantaneous signalling.

2.1 Kinematics and Dynamics

This might seem problematic, it still being common among physicists to describe superluminal effects as ‘acausal’. But superluminal signalling violates causality – that is, gives rise to backwards-in-time signals in some frames – *if* one assumes a locally Minkowski structure for spacetime. Historically, the Minkowski structure was developed for a local physics. If Nature turns out to be nonlocal, then one should consider revising that structure.

This may seem an obvious point. Yet, many physicists tend to think of Minkowski spacetime as a prior (‘God-given’) background or stage on which physics takes place (at least locally, ignoring gravitation for the moment). A common view is that laws such as Maxwell’s equations possess Lorentz symmetry ‘because’ spacetime has a Minkowski structure. It is as if we were first given the stage of spacetime, and afterwards we wrote laws on it. But one could

equally take the view that spacetime has a Minkowski structure ‘because’ the known laws all have a Lorentz symmetry.² This would certainly be closer to the historical facts: first one discovers certain symmetries in the behaviour of matter, then one postulates a spacetime structure that incorporates those symmetries.

From this last perspective, one should be open to the possibility that, in the future, new phenomena might break old symmetries, or, that new symmetries might emerge; and in either case, the structure of spacetime might have to be revised. In a word, one should bear in mind that new laws of physics might demand a new structure for spacetime.

Kinematics and dynamics are two sides of the same coin (Brown 2005). As we discover new dynamical effects, we should be prepared to modify our kinematics (or spacetime geometry) if necessary or convenient. In section 3 we shall describe an effect whose observation in the future is, in the author’s opinion, to be expected from a hidden-variables perspective, and which would, we argue, lead us to modify our current relativistic kinematics.

The rise of relativity theory should have taught us the lesson that the structure of spacetime is not *a priori*, but depends on physics – just as more recently, with the rise of quantum computing, we have come to learn that the theory of computation is not *a priori* but depends on physics. Unfortunately, after 1905, the dogma of Newtonian spacetime was quickly replaced by the dogma of (local) Minkowski spacetime.

The replacement of one rigid view by another was perhaps due in part to Einstein’s unfortunate ‘operational’ presentation in his first relativity paper of 1905 (Einstein 1905), which treated macroscopic rods and clocks as if they were fundamental entities. This led to a widespread misunderstanding, according to which the resulting kinematics was somehow logically inevitable, when in fact it was highly contingent on properties of the physical dynamics known at the time.

2.2 Einstein and Poincaré in 1905

Einstein himself acknowledged the conceptual mistake in his autobiographical notes of 1949:

‘The theory [special relativity] introduces two kinds of physical things, i.e., (1) measuring rods and clocks, (2) all other things, e.g., the electro-magnetic field, the material point, etc. This, in a certain sense, is inconsistent; strictly speaking measuring rods and clocks would have to be represented as solutions of the basic equations (objects consisting of moving atomic configurations), not, as it were, as theoretically self-sufficient entities’ (Einstein 1949b).

In 1905 Einstein had treated rods and clocks as primitive entities, independent of theory (‘theoretically self-sufficient’). But in fact, as Einstein later

²Arguably, these are two different ways of saying the same thing. The kinematical structure of spacetime cannot be disentangled from the dynamics taking place within it (Brown 2005).

recognised, rods and clocks are phenomenological entities arising out of some underlying theory (perhaps involving particles and/or fields). In reality, we need some body of theory to tell us how to construct reliable rods and clocks and to analyse their behaviour. For example, using theory we can calculate the effect of acceleration on a real clock, and so use theory to design more robust clocks. Rods and clocks are not simply ‘given’ to us.

The modern view of special relativity, used in high-energy physics for example, makes no mention of rods and clocks. It concerns particles and fields on Minkowski spacetime. The essence of Lorentz invariance is simply that the Lagrangian density appearing in quantum field theory should be a Lorentz scalar (resulting in a Lorentz-covariant S -matrix). Nor do classical light waves play any special role: what matters are the symmetries of the fundamental equations, not the speed of propagation of some particular particle or field. After all, the photon might turn out to have a small mass. That we first discovered Lorentz invariance via the classical electromagnetic field is merely a historical accident, and Einstein’s 1905 approach – based on macroscopic rods, clocks and classical light waves – is merely a historical (and fundamentally inconsistent) heuristic.

The popularity of Einstein’s ‘operational’ approach to special relativity had the effect of introducing a deep and widespread confusion between phenomenological and fundamental entities. This confusion seems to have encouraged an overly-rigid philosophy of space and time, in which Einstein’s kinematics came to appear as an inevitable – *a priori*, and theoretically self-sufficient – background to the laws of dynamics.³ Today, despite the discovery of quantum nonlocality, there is still a reluctance in some quarters to even consider changing our view of spacetime structure.

It is often claimed that Einstein’s 1905 approach should be regarded as not merely a historical curiosity, but as the proper way to understand special relativity. After all, it was this approach which in fact first led us to special relativity. And how else could special relativity have been discovered? But as a matter of historical fact, building on earlier work by Lorentz and others, the formal structure of special relativity – the relativity principle, the universality of the Lorentz group, the relativistic addition of velocities, and even 4-vectors with the associated 4-dimensional invariant interval (later taken up by Minkowski in 1908) – was independently arrived at by Poincaré in his paper ‘On the Dynamics of the Electron’ (1906). This paper was submitted to a mathematical journal in Palermo, in the same summer (of 1905) as Einstein’s first relativity paper was submitted to the *Annalen der Physik*; it was published in 1906. A summary of the results was published in 1905, in a short paper of the same title (Poincaré 1905b).⁴

³As we shall discuss elsewhere, the confusion between phenomenology and fundamentals also led to inconsistencies in quantum theory, in the form of the ‘measurement’ or ‘reality’ problem.

⁴According to its original title page, Poincaré’s long paper ‘On the Dynamics of the Electron’ (1906) was accepted for publication by the *Rendiconti del Circolo Matematico di Palermo* on 23 July 1905, printed on 14 December, and officially published in 1906. The short summary with the same title was (according to Pais; 1982) communicated to the Académie des Sciences

The importance of Poincaré's 'Palermo' paper has been underestimated, even by some historians. Certainly, most physicists are not even aware of its existence. (An incomplete translation appears in Kilmister (1970); a modernised presentation of most of the paper is given in Schwartz (1971, 1972). For detailed analyses of the paper, see Miller (1973) and Zahar (1989). More recent discussions of Poincaré's extensive contributions to special relativity have been given by Darrigol (1995, 1996) and Granek (2000).)

Among physicists, Pauli was exceptional in being careful to credit Poincaré's Palermo paper properly throughout his celebrated treatise on relativity (Pauli 1958; first published in 1921). For example, with reference to the Palermo paper, Pauli notes that:

‘The formal gaps left by Lorentz's work were filled by Poincaré. He stated the relativity principle to be generally and rigorously valid. Since he assumed Maxwell's equations to hold for the vacuum, this amounted to the requirement that all laws of nature must be covariant with respect to the ‘Lorentz transformation’. The terms ‘Lorentz transformation’ and ‘Lorentz group’ occurred for the first time in this paper by Poincaré.⁵ Poincaré further corrected Lorentz's formulae for the transformations of charge density and current and so derived the complete covariance of the field equations of electron theory’ (Pauli 1958: 3).

Pauli correctly credits Poincaré, not only for postulating the Lorentz group as a universal symmetry group, but also for the first use of 4-vectors and of the associated 4-dimensional invariant interval. Pauli writes:

‘As a precursor of Minkowski one should mention Poincaré He already introduced on occasion the imaginary coordinate $u = ict$ and combined, and interpreted as point coordinates in R_4 , those quantities which we now call vector components. Furthermore, the invariant interval plays a rôle in his considerations’ (Pauli 1958: 21).

How had Poincaré done it? The answer is, along the lines that most workers in high-energy physics would probably take today. (Poincaré was concerned with the detailed structure and dynamics of the electron, the ‘elementary particle physics’ of the time.) He first notes the experimental failure to detect the absolute motion of the Earth, and proposes that this is ‘a general law of Nature’, which he calls the ‘Relativity Postulate’ (Poincaré 1906: 129). Further, following and perfecting the extensive work of Lorentz, Poincaré notes that Maxwell's equations have the Lorentz group as an exact symmetry group, and *postulates* that this is a universal symmetry applicable to all forces (including gravitation). Poincaré recognises that this postulate suffices to explain the observed invariance of phenomena under a boost. Citing Lorentz, Poincaré writes:

in Paris on 5 June 1905; it was published in 1905 in the *Comptes Rendus de l'Académie des Sciences de Paris* (Poincaré 1905b). Einstein's first relativity paper was received by the *Annalen der Physik* on 30 June 1905 and published in 1905 (Einstein 1905).

⁵This sentence appears as a footnote in the original text.

‘If one can impart a common boost to the whole system without any of the apparent phenomena being modified, this is because the equations of an electromagnetic medium are not changed by certain transformations, which we shall call *Lorentz transformations*; two systems, one at rest, the other in motion, thus become exact images of each other. According to him [Lorentz], all forces, whatever their origin, are affected by the Lorentz transformation (and therefore by a boost) in the same manner as electromagnetic forces’ (Poincaré 1906: 130; translation by the author).

Poincaré then deduces the detailed structure of the Lorentz group, including the relativistic addition of velocities, noting that the group leaves invariant the quadratic form $x^2 + y^2 + z^2 - t^2$. There follows an extensive discussion of relativistic electron dynamics. In the final section of the paper, Poincaré formulates a Lorentz-covariant generalisation of Newtonian gravitation, with gravitational interactions propagating at the speed of light⁶. This last theory is formulated by finding Lorentz-invariant functions of the velocities and relative positions of the masses (as well as of time). To find these, Poincaré uses the fact that the Lorentz group may be regarded as the group of rotations in a 4-dimensional space with coordinates x, y, z, it . As Poincaré put it:

‘We see that the Lorentz transformation is nothing but a rotation of this space around the origin’ (Poincaré 1906: 168; translation by the author).

Independently of Einstein and Minkowski, then, in 1905 Poincaré arrived at the formal, mathematical structure of Minkowski spacetime and the Lorentz group.

One may argue over the extent to which Poincaré understood the new kinematics defined by his formalism. According to Darrigol (1995: 35, 1996: 280), Poincaré did understand that the Lorentz-transformed coordinates were to be identified with the actual readings of boosted rods and clocks, since he regarded Lorentz invariance as a physical (not just a mathematical) symmetry, whereby ‘apparent phenomena’ in a moving system follow the same laws as phenomena in a system at rest. Similarly, according to Janssen and Stachel (2004): ‘Unlike Lorentz, Poincaré realized that the auxiliary quantities are the measured quantities for the moving observer’. In fact, as early as 1900, Poincaré understood that if experimenters moving with speed v were to assume that the speed of light is c in every direction, then (to lowest order in v/c) they would synchronise clocks separated by a distance x such that the settings differ by $-vx/c^2$ (see section 4.1). At least to lowest order in v/c , Poincaré had already understood in 1900 that the Lorentz-transformed time corresponded to the actual readings of moving clocks.⁷

⁶Poincaré’s short summary (1905b) refers to ‘gravitational waves’ propagating between gravitating bodies. For a detailed discussion of Poincaré’s 1905 theory of gravity, see Zahar (1989: 192–200).

⁷Brown (2005), however, questions whether in 1905 Poincaré fully understood the physical significance of the transformed coordinates to higher orders in v/c . On the other hand,

Any suggestion that Poincaré viewed the Lorentz transformation as a purely mathematical change of variables seems untenable. After all, Poincaré asserted that Lorentz invariance alone sufficed to explain the invariance of apparent phenomena under a boost, so the transformed quantities in question must indeed have been regarded as those measured by a moving observer. (In contrast, for Lorentz, his ‘theorem of corresponding states’ – which was mathematically almost the same as Lorentz invariance – had to be supplemented by further physical assumptions to explain the failure to detect ether drift (Janssen and Stachel 2004).) Further, in his Palermo paper, Poincaré derives real physical corrections to Newton’s law of gravity, from the requirement that the law of motion for gravitating bodies should be covariant with respect to rotations in what we would now call Minkowski space (with coordinates x, y, z, it). For Poincaré, this symmetry clearly had real, observable physical consequences.

One may also ask if Poincaré (like Lorentz) took the view that there was a true rest frame. According to Darrigol (1995: 40), for example, Poincaré did indeed maintain this view (which Darrigol sees as the only essential difference between Poincaré and Einstein in 1905). On this point it should be remembered that (as we shall discuss in section 5) for Poincaré, the geometry of spacetime is not a fact about the world but merely a convenient convention, so that if one finds it convenient one may indeed think in terms of absolute space and time.⁸ In any case, this interpretation of Poincaré’s made no empirical difference. Further, we argue that in the light of quantum nonlocality it may well be the better interpretation after all.

It does seem fair to say – despite (limited) anticipations by Fitzgerald, Lorentz, and Larmor⁹ – that a clear and complete statement of universal time dilation and length contraction is first found in Einstein’s paper of 1905. Poincaré’s Palermo paper discusses length contraction for spherical electrons, but does not explicitly mention time dilation, despite extensive use of the Lorentz-transformed time variable. As Pauli observed, regarding time dilation:

‘While this consequence of the Lorentz transformation was already implicitly contained in Lorentz’s and Poincaré’s results, it received its first clear statement only by Einstein’ (Pauli 1958: 13).

It was claimed by Pais (1982: 164, 167–168) that even after 1905 Poincaré did not understand special relativity, because, judging from the text of his lectures at Göttingen in 1909 (Poincaré 1910), he did not understand that length

Darrigol (1995: 37–40) shows that, in lectures delivered at the Sorbonne in 1906–07, Poincaré (apparently independently of Einstein) generalised his 1900 discussion of clock synchronisation (taking into account length contraction) to obtain the full Lorentz-transformed time to all orders in v/c .

⁸We are inclined to agree with Zahar (1989: 150): ‘.... that Poincaré did discover special relativity, that his philosophy of science provided him with heuristic guidelines, but that certain ambiguities within that same philosophy prevented both his contemporaries and many historians from appreciating the true value of his contribution.’

⁹A limited form of time dilation was anticipated by Larmor (in a paper of 1897, and in his book of 1900), and by Lorentz (in a paper of 1899). See Brown (2005: section 4.5), and Janssen and Stachel (2004).

contraction was a consequence of Einstein's two postulates (the relativity principle and the light postulate), but instead insisted on including length contraction as a third postulate. In the author's opinion, this issue is confused because Einstein's 1905 approach actually contains an implicit third postulate: that under a boost from one rest frame to another, unit rods are transformed into unit rods, and similarly for unit clock ticks. Einstein himself admitted this, in a footnote to a review he published in 1910, where he writes:

‘It should be noted that we will always implicitly assume that the fact of a measuring rod or a clock being set in motion or brought to rest does not change the length of the rod or the rate of the clock’ (Einstein 1993: 130).

To the author's knowledge, the only other place in the historical literature where Einstein's implicit third postulate is mentioned is in Born's relativity text (1962).¹⁰ In fact, Born discusses this postulate in some detail, and regards it as of crucial importance. He writes:

‘.... it is assumed as self-evident that a measuring rod which is brought into one system of reference S and then into another S' under exactly the same physical conditions would represent the same length in each A fixed rod that is at rest in the system S and is of length 1 cm. will, of course, also have the length 1 cm. when it is at rest in the system S' Exactly the same would be postulated for the clocks We might call this tacit assumption of Einstein's theory the “principle of the physical identity of the units of measure” This is the feature of Einstein's theory by which it rises above the standpoint of a mere convention and asserts definite properties of real bodies’ (Born 1962: 251–252).

It might be thought that the third postulate could be dispensed with, by using the relativity principle to deduce that any specific process for constructing rods and clocks must give the same results in all inertial frames. Certainly, using the light postulate as well, one could then deduce that the Lorentz transformation relates the readings of *different* rods and clocks that have been constructed (by a similar process) in different inertial frames. However, one would still have deduced nothing about what happens when the *same* rod or clock is boosted (or accelerated) from one inertial frame to another. (As an example one might, in principle, envisage a theory satisfying the relativity principle and the light postulate, but with the additional property that once a rod or clock has been constructed in a given inertial frame it is destroyed by any subsequent arbitrarily small acceleration.)

Thus, despite widespread opinion to the contrary, length contraction and time dilation under a boost do *not* follow from Einstein's two postulates alone.

¹⁰The recent book by Brown (2005: section 2.4) calls this assumption the ‘boostability’ of rods and clocks, and regards it more as a ‘stipulation’ (or convenient convention) than an assumption.

A further postulate is required, to relate the readings of rods and clocks boosted from one inertial frame to another.¹¹

In view of the crucial importance of the third assumption implicitly used by Einstein, it must be regarded as regrettable that Einstein did not mention it explicitly in his first relativity paper. In the author's opinion, it is quite possible that Poincaré was aware of this lacuna, explaining why in his lectures of 1909 (Poincaré 1910) – where he sketches an axiomatic basis for the ‘new mechanics’, in terms of simple physical postulates independent of the details of Maxwell's equations, much as Einstein did in 1905 – he added the third postulate of length contraction, which was not as elegant as the third postulate implicitly used by Einstein, but effective nonetheless.¹²

In any case, such detailed questions of priority, or of who understood exactly what and when, while historically interesting, are not strictly relevant here. What really matters, for our purpose, is that the approach taken in Poincaré's Palermo paper – in which the Lorentz group is first discovered through Maxwell's equations and then postulated to be a universal (physical) symmetry group – quite plainly *could have been* the historical route to special relativity. Regardless of the extent to which Poincaré did or did not understand it at the time, the fact is that the kinematics of Minkowski spacetime was contained in the formal structure put forward in Poincaré's paper.

Minkowski, in his famous lecture on ‘Space and Time’ delivered in 1908 (Minkowski 1952), appears to express a preference for this sort of approach, which actually goes back to 1887 when Voigt (1887) derived the Lorentz transformation, up to an overall constant factor, as a symmetry of the (scalar) wave equation.¹³ According to Minkowski:

‘Now the impulse and true motive for assuming the group G_c [that is, the Poincaré group, which leaves invariant the 4-dimensional interval] came from the fact that the differential equation for the propagation of light in empty space possesses that group G_c . An application of this fact in its essentials has already been given by W. Voigt, Göttinger Nachrichten, 1887, p. 41¹⁴’ (Minkowski 1952: 81).

It is sometimes argued that Einstein's operational approach has the advantage of being independent of the details of specific equations such as Maxwell's. This may be so, but Einstein's approach also has the disadvantage of giving a special status to classical light waves, and of being conceptually inconsistent with regard to the nature of rods and clocks. As for Poincaré's approach, as

¹¹In fact, a still further assumption of spatial isotropy is also needed – see Brown (2005: section 5.4.3).

¹²As noted by Darrigol (1995: 39), Poincaré had also used length contraction as a hypothesis in his Sorbonne lectures of 1906-07. Again, in the author's view, Poincaré may well have understood that some such extra hypothesis was needed to relate measurements in different frames.

¹³Voigt's paper is briefly discussed by Pais (1982: 121–122), and in great detail by Ernst and Hsu (2001), who also provide an English translation of it.

¹⁴This sentence appears as a footnote in the original text.

a scientific methodology there is nothing wrong with discovering a symmetry in certain equations and then postulating that the symmetry is universal (regardless of whether those equations turn out to be fundamental or not). This is, after all, common practice in high-energy physics today. Clearly, Einstein's operational approach was not necessary, and special relativity could have been (and arguably essentially was) discovered without appeal to a fundamentally inconsistent operationalism.

In the author's opinion, if Poincaré's approach had in fact been the generally-accepted historical route to special relativity, then physicists today might be more keenly aware that spacetime geometry is not 'prior to' dynamics but rather a reflection of symmetries of the currently-known dynamics.¹⁵ From this standpoint, as physics progresses, the structure of spacetime is as subject to possible revision as are the laws of dynamics themselves.

3 Instantaneous Signalling in Quantum Non-Equilibrium

In this section we show how, in deterministic hidden-variables theories, a non-standard distribution of hidden variables (generically) gives rise to instantaneous signalling at the statistical level. We first discuss this for general theories (Valentini 2002a,b), then for the specific example of the pilot-wave theory of de Broglie and Bohm (Valentini 1991b, 2002c).

3.1 General (Deterministic) Hidden-Variables Theories

For a 2-state system, consider quantum observables of the form $\hat{\sigma} = \mathbf{m} \cdot \hat{\sigma}$, where \mathbf{m} is a unit vector specifying a point on the Bloch sphere and $\hat{\sigma}$ is the Pauli spin operator. The values $\sigma = \pm 1$ are obtained upon performing a quantum measurement of $\hat{\sigma}$. Over an ensemble with density operator $\hat{\rho}$, the quantum expectation value of $\hat{\sigma}$ is given by the Born rule as $\langle \hat{\sigma} \rangle = \text{Tr}(\hat{\rho} \mathbf{m} \cdot \hat{\sigma}) = \mathbf{m} \cdot \mathbf{P}$, where $\mathbf{P} = \langle \hat{\sigma} \rangle$ (with norm $0 \leq P \leq 1$) is the mean polarisation. The quantum probabilities $p_{\text{QT}}^{\pm}(\mathbf{m})$ for outcomes $\sigma = \pm 1$ are then fixed as

$$p_{\text{QT}}^{\pm}(\mathbf{m}) = \frac{1}{2} (1 \pm \mathbf{m} \cdot \mathbf{P}) \quad (1)$$

In a (deterministic) hidden-variables theory, for every run of the experiment with measurement axis \mathbf{m} , there are hidden parameters collectively denoted λ that determine the outcome $\sigma = \pm 1$ according to some mapping $\sigma = \sigma(\mathbf{m}, \lambda)$. Over an ensemble of experiments, the observed distribution of outcomes is explained by some assumed distribution $\rho_{\text{QT}}(\lambda)$ of parameters λ , where $\rho_{\text{QT}}(\lambda)$ is such that expectations

$$\langle \sigma(\mathbf{m}, \lambda) \rangle_{\text{QT}} = \int d\lambda \rho_{\text{QT}}(\lambda) \sigma(\mathbf{m}, \lambda)$$

¹⁵Even if Poincaré himself, for philosophical reasons of his own, seemed to prefer retaining the old notions of space and time in the background.

agree with the quantum prediction $\langle \mathbf{m} \cdot \hat{\sigma} \rangle$. The values of λ are usually defined at some initial time, say at the time of preparation of the quantum state. The outcomes $\sigma = \sigma(\mathbf{m}, \lambda)$ are defined at the time of measurement.

Now, there is a clear conceptual distinction between the initial values λ and the mapping $\sigma = \sigma(\mathbf{m}, \lambda)$ to final outcomes σ . In particular, the former amount to what are usually called ‘initial conditions’, while the latter would usually be called a ‘dynamical law’ that maps initial conditions to final states. Therefore, once such a theory has been constructed, one may contemplate arbitrary initial conditions – over an ensemble, distributions $\rho(\lambda) \neq \rho_{QT}(\lambda)$ – while retaining the mapping $\sigma = \sigma(\mathbf{m}, \lambda)$. Generically, such ‘non-quantum’ or ‘non-equilibrium’ distributions will yield expectation values

$$\langle \sigma(\mathbf{m}, \lambda) \rangle = \int d\lambda \rho(\lambda) \sigma(\mathbf{m}, \lambda)$$

that disagree with quantum theory, and the statistics of outcomes will generally violate the standard quantum-theoretical constraints. Note the key conceptual point: we have the same deterministic mapping $\sigma = \sigma(\mathbf{m}, \lambda)$ for each system, regardless of the (equilibrium or non-equilibrium) distribution for the ensemble.

Many of the supposedly fundamental constraints of quantum theory, such as statistical locality, are (from a hidden-variables perspective) merely contingent features of the special distribution $\rho_{QT}(\lambda)$. As noted in section 1.3, there is an analogy here with the contingent constraints that arise in classical physics in a state of global thermal equilibrium: the inability to convert heat into work is not fundamental, but a contingency due to all systems having the same temperature.

Consider a pair of widely-separated 2-state systems with spatial locations A and B . Quantum measurements of $\hat{\sigma}_A \equiv \mathbf{m}_A \cdot \hat{\sigma}_A$, $\hat{\sigma}_B \equiv \mathbf{m}_B \cdot \hat{\sigma}_B$ can yield outcomes $\sigma_A, \sigma_B = \pm 1$. For the singlet state

$$|\Psi\rangle = (|+\mathbf{n}, -\mathbf{n}\rangle - |-\mathbf{n}, +\mathbf{n}\rangle) / \sqrt{2}$$

(for any axis \mathbf{n}) quantum theory predicts that outcomes $\sigma_A, \sigma_B = \pm 1$ occur in the ratio 1 : 1 at each wing, with a correlation

$$\langle \Psi | \hat{\sigma}_A \hat{\sigma}_B | \Psi \rangle = -\mathbf{m}_A \cdot \mathbf{m}_B \quad (2)$$

Nevertheless, the distant settings have no effect on the expectation values ($\langle \hat{\sigma}_{A,B} \rangle = 0$) or on the probabilities ($p_{QT}^{\pm}(\mathbf{m}_{A,B}) = 1/2$) at each wing, making nonlocal signalling impossible.

However, from a hidden-variables perspective, Bell’s theorem (1964) tells us that to reproduce this correlation a hidden-variables theory must take the nonlocal form

$$\sigma_A = \sigma_A(\mathbf{m}_A, \mathbf{m}_B, \lambda), \quad \sigma_B = \sigma_B(\mathbf{m}_A, \mathbf{m}_B, \lambda) \quad (3)$$

in which the individual outcomes σ_A, σ_B do depend on the distant measurement settings. Only with such nonlocal dependence can the theory reproduce the quantum correlation

$$\langle \sigma_A \sigma_B \rangle_{QT} \equiv \int d\lambda \rho_{QT}(\lambda) \sigma_A(\mathbf{m}_A, \mathbf{m}_B, \lambda) \sigma_B(\mathbf{m}_A, \mathbf{m}_B, \lambda) = -\mathbf{m}_A \cdot \mathbf{m}_B \quad (4)$$

for some ensemble distribution $\rho_{QT}(\lambda)$. More precisely, at least one of σ_A, σ_B must depend on the distant setting, and without loss of generality we shall assume that σ_A has a nonlocal dependence on \mathbf{m}_B .

Now, for an arbitrary ensemble with $\rho(\lambda) \neq \rho_{QT}(\lambda)$, in general

$$\langle \sigma_A \sigma_B \rangle \equiv \int d\lambda \rho(\lambda) \sigma_A(\mathbf{m}_A, \mathbf{m}_B, \lambda) \sigma_B(\mathbf{m}_A, \mathbf{m}_B, \lambda) \neq -\mathbf{m}_A \cdot \mathbf{m}_B \quad (5)$$

and the outcomes $\sigma_A, \sigma_B = \pm 1$ at each wing will occur in a ratio generally differing from 1 : 1. Further, under a change in the measurement setting at one wing, the outcome statistics at the distant wing will generally change, amounting to a nonlocal signal at the statistical level. The key point here is that, assuming a nonlocal dependence of σ_A on \mathbf{m}_B , the ‘transition sets’

$$\begin{aligned} T_A(-, +) &\equiv \{\lambda | \sigma_A(\mathbf{m}_A, \mathbf{m}_B, \lambda) = -1, \sigma_A(\mathbf{m}_A, \mathbf{m}'_B, \lambda) = +1\} \\ T_A(+, -) &\equiv \{\lambda | \sigma_A(\mathbf{m}_A, \mathbf{m}_B, \lambda) = +1, \sigma_A(\mathbf{m}_A, \mathbf{m}'_B, \lambda) = -1\} \end{aligned}$$

cannot be empty for arbitrary settings $\mathbf{m}_A, \mathbf{m}_B, \mathbf{m}'_B$. Some outcomes at A must change under a shift $\mathbf{m}_B \rightarrow \mathbf{m}'_B$ at B . In quantum equilibrium, the ratio of outcomes $\sigma_A = \pm 1$ is 1 : 1 for all settings, therefore we must have ‘detailed balancing’

$$\mu_{QT}[T_A(-, +)] = \mu_{QT}[T_A(+, -)]$$

with respect to the equilibrium measure $d\mu_{QT} \equiv \rho_{QT}(\lambda)d\lambda$. In other words, in quantum equilibrium, the fraction of the ensemble making the transition $\sigma_A = -1 \rightarrow \sigma_A = +1$ under $\mathbf{m}_B \rightarrow \mathbf{m}'_B$ must equal the fraction making the reverse transition $\sigma_A = +1 \rightarrow \sigma_A = -1$. (This is analogous to the principle of detailed balance in statistical mechanics: thermal equilibrium is maintained if the mean transition rate from state i to state j is equal to the mean transition rate from j to i .) Since $T_A(-, +)$ and $T_A(+, -)$ are fixed by deterministic equations, they are independent of the ensemble distribution of λ . Thus, for a hypothetical non-equilibrium ensemble $\rho(\lambda) \neq \rho_{QT}(\lambda)$, in general

$$\mu[T_A(-, +)] \neq \mu[T_A(+, -)]$$

where $d\mu \equiv \rho(\lambda)d\lambda$. In other words, the fraction of the non-equilibrium ensemble making the transition $\sigma_A = -1 \rightarrow \sigma_A = +1$ will not in general balance the fraction making the reverse transition; the ratio of outcomes at A will in general change under $\mathbf{m}_B \rightarrow \mathbf{m}'_B$ and there will be instantaneous signals at the statistical level from B to A (Valentini 2002a,b).

In any deterministic hidden-variables theory, then, hypothetical non-equilibrium distributions $\rho(\lambda) \neq \rho_{QT}(\lambda)$ generally make it possible to use nonlocality for instantaneous signalling (just as, in ordinary statistical physics, differences of temperature make it possible to convert heat into work) (Valentini 2002a,b).

3.2 The Example of Pilot-Wave Theory

Non-equilibrium signalling at a distance was first noted (Valentini 1991b, 2002c) in the hidden-variables theory of de Broglie and Bohm (de Broglie 1928; Bohm

1952a,b). In this ‘pilot-wave theory’ (as it was originally called by de Broglie), a system with wave function $\Psi(X, t)$ satisfying the Schrödinger equation

$$i\frac{\partial\Psi}{\partial t} = \hat{H}\Psi \quad (6)$$

has an actual configuration $X(t)$ whose motion is given by the first-order differential equation

$$\dot{X}(t) = \frac{J(X, t)}{|\Psi(X, t)|^2} \quad (7)$$

where $J = J[\Psi] = J(X, t)$ (which in quantum theory is called the ‘probability current’) satisfies the continuity equation

$$\frac{\partial|\Psi|^2}{\partial t} + \nabla_X \cdot J = 0 \quad (8)$$

(which follows from (6)). In pilot-wave theory, Ψ is regarded as an objective physical field guiding the system.

For example, for a system of N particles with 3-vector positions $\mathbf{x}_i(t)$ and masses m_i ($i = 1, 2, \dots, N$) the wave function $\Psi(X, t)$ on $3N$ -dimensional configuration space ($X \equiv (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N)$) is a complex field obeying the Schrödinger equation

$$i\frac{\partial\Psi}{\partial t} = \sum_{i=1}^N -\frac{1}{2m_i} \nabla_i^2 \Psi + V\Psi \quad (9)$$

and the particle velocities are given by

$$\frac{d\mathbf{x}_i}{dt} = \frac{1}{m_i} \text{Im} \left(\frac{\nabla_i \Psi}{\Psi} \right) = \frac{\nabla_i S}{m_i} \quad (10)$$

where $\Psi = |\Psi| e^{iS}$ and we take $\hbar = 1$.

Equations (6) and (7) determine the motion $X(t)$ of an *individual* system, given the initial configuration $X(0)$ and wave function $\Psi(X, 0)$ at $t = 0$. If we are given an arbitrary initial distribution $P(X, 0)$, for an ensemble of systems with the same wavefunction $\Psi(X, 0)$, then the evolution of $P(X, t)$ is necessarily given by the continuity equation

$$\frac{\partial P}{\partial t} + \nabla_X \cdot (P\dot{X}) = 0 \quad (11)$$

This same equation is satisfied by $|\Psi|^2$, as follows from (8). Thus, if $P(X, 0) = |\Psi(X, 0)|^2$ at some initial time, then $P(X, t) = |\Psi(X, t)|^2$ at all times t . As shown by Bohm (1952a,b), one then recovers the statistical predictions of quantum theory.

In pilot-wave theory, the outcome obtained in a given experiment is determined by $X(0)$ and $\Psi(X, 0)$, so that one may identify λ with the pair $X(0), \Psi(X, 0)$. For an ensemble of experiments with the same $\Psi(X, 0)$, in effect λ is

just $X(0)$, and the distribution $\rho_{QT}(\lambda)$ is given by $P_{QT}(X, t) = |\Psi(X, t)|^2$. As in the general discussion above, we may retain the same deterministic dynamics for individual systems, and consider a non-standard distribution of initial conditions. Here, this means we retain the dynamical equations (6), (7) and consider an arbitrary initial ensemble with $P(X, 0) \neq |\Psi(X, 0)|^2$. The evolution of $P(X, t)$ will be determined by (11).

In appropriate circumstances, (11) leads to relaxation $P \rightarrow |\Psi|^2$ on a coarse-grained level (Valentini 1991a, 1992, 2001; Valentini and Westman 2005), much as the corresponding classical evolution on phase space leads to thermal relaxation. However, for as long as the ensemble is in non-equilibrium, the statistics of outcomes of quantum measurements will disagree with quantum theory.

As required by Bell's theorem, pilot-wave theory is fundamentally nonlocal. For two particles whose wave function $\Psi(\mathbf{x}_A, \mathbf{x}_B, t)$ is entangled, $\dot{\mathbf{x}}_A(t) = \nabla_A S(\mathbf{x}_A, \mathbf{x}_B, t)/m_A$ depends instantaneously on \mathbf{x}_B , and ordinary operations on particle B – such as switching on a local potential – have an instantaneous effect on the motion of particle A . But for a quantum equilibrium ensemble $P(\mathbf{x}_A, \mathbf{x}_B, t) = |\Psi(\mathbf{x}_A, \mathbf{x}_B, t)|^2$, such operations on particle B have no statistical effect on particle A : the individual nonlocal effects are masked by quantum noise.

As in the general case discussed above, nonlocality is (generally speaking) hidden by statistical noise only in quantum equilibrium. For an ensemble of entangled particles with initial distribution $P(\mathbf{x}_A, \mathbf{x}_B, 0) \neq |\Psi(\mathbf{x}_A, \mathbf{x}_B, 0)|^2$, a local change in the Hamiltonian of particle B generally induces an instantaneous change in the marginal distribution $p_A(\mathbf{x}_A, t) \equiv \int d^3\mathbf{x}_B P(\mathbf{x}_A, \mathbf{x}_B, t)$ of particle A . For example, in one dimension, a sudden change $\hat{H}_B \rightarrow \hat{H}'_B$ in the Hamiltonian of particle B induces a change $\Delta p_A \equiv p_A(x_A, t) - p_A(x_A, 0)$ of the form (for small t) (Valentini 1991b)

$$\Delta p_A = -\frac{t^2}{4m} \frac{\partial}{\partial x_A} \left(a(x_A) \int dx_B b(x_B) \frac{P(x_A, x_B, 0) - |\Psi(x_A, x_B, 0)|^2}{|\Psi(x_A, x_B, 0)|^2} \right) \quad (12)$$

(Here $m_A = m_B = m$, the factor $a(x_A)$ depends on $\Psi(x_A, x_B, 0)$, while $b(x_B)$ also depends on \hat{H}'_B and vanishes if $\hat{H}'_B = \hat{H}_B$.) In general, the signal is non-zero if $P_0 \neq |\Psi_0|^2$ (that is, if $\rho(\lambda) \neq \rho_{QT}(\lambda)$).

Elsewhere (Valentini 2002c), using the example of pilot-wave theory, we have described how non-equilibrium particles might be detected in practice, by the statistical analysis of random samples (taken, for example, from a parent population of relic particles left over from the early universe). Once such particles have been identified, they may be used as a resource for superluminal signalling; further, they may be used to perform 'subquantum measurements' on ordinary, equilibrium systems (Valentini 2002c).

4 Absolute Simultaneity in Flat and Curved Spacetime

We have seen that, even at ordinary laboratory distances and energies, quantum non-equilibrium would unleash instantaneous signals between entangled systems. This raises the question of how these signals could mesh with the surrounding approximately classical spacetime. As we emphasised in the Introduction, this question must have an answer, *irrespective* of the underlying microscopic theory of spacetime.

If experimenters at spacetime events A and B had access to non-equilibrium systems entangled between A and B , then they would be able to signal back and forth to each other instantaneously. In an arbitrarily short time (as measured at each wing), a long conversation could in principle take place, during which (for example) the experimenters agree to set their clocks to read time $t = 0$. They could signal to each other to confirm that they have done so. In such conditions, A and B must be regarded as simultaneous events, and the agreed-upon time variable t would define an absolute simultaneity. Thus, using non-equilibrium matter, experimenters at remote locations could set their clocks to read the same instantaneous time t .

There are, however, some differences depending on whether gravitation is absent or present. Let us discuss these in turn.

4.1 Flat Spacetime

In the absence of gravitation (where the kinematics is usually represented by flat Minkowski spacetime), remote experimenters may use entangled non-equilibrium systems to set their clocks to read the same time t . However, they must be careful to bear in mind that clocks in motion drift out of synchronisation with clocks at rest. For if a clock undergoes a spatial displacement $d\mathbf{x}$ in a time dt , then the ‘proper’ time $d\tau$ ticked by the clock is given by

$$d\tau^2 = dt^2 - d\mathbf{x}^2$$

Thus, a clock moving through space with speed $v = |d\mathbf{x}/dt|$ is slowed by the factor $1/\sqrt{1 - v^2/c^2}$. Here, this ‘time dilation’ may be regarded as a dynamical effect of motion on the rate of evolution of physical systems, as originally anticipated by Larmor and Lorentz. (An instructive account of this viewpoint was given by Bell (1987: 67–80).)

One must then distinguish between *simultaneity* and *synchronicity*. The first refers to events that exist ‘in unison’, in a sense that could be verified by nonlocal communication. The second refers merely to the coincidence of readings of certain (usually classical, macroscopic) systems called ‘clocks’, where for dynamical reasons the rate of evolution of such systems depends on how fast they are moving through space. Simultaneity is not equivalent to synchronicity.

For example, if two clocks are initially close together and synchronised in a standard inertial frame with time function t , and if one clock is accelerated

and eventually returns close to its partner, then finally the two clocks will be out of step, as the accelerated clock will have been slowed down. If t coincides with our absolute time, the final clock readings will correspond to simultaneous events, yet, the readings will not be synchronous.

Note that this dynamical effect of motion occurs at the classical macroscopic level, as well as at the statistical level for ensembles of microscopic quantum systems, but it is not necessarily relevant to the deeper level of hidden variables. (For example, decay rates for individual atoms are affected by time dilation, but such rates apply to quantum ensemble averages and not to individual systems.) Therefore, there is no reason why this dynamical effect should be built into the fundamental kinematics (as it usually is).

The objection might be raised that superluminal signals in a given frame would ‘violate causality’, since in other frames the signals could travel backwards in time, leading to paradoxes. But as we discussed in section 2, this argument assumes that the structure of spacetime is fundamentally Minkowskian. There is no reason to assume this. At the nonlocal hidden-variable level, there may well be a preferred slicing of spacetime, with a time function t that defines a fundamental causal sequence (Popper 1982; Bohm and Hiley 1984; Bell 1986, 1987).

Clearly, in a given preferred frame with standard Lorentzian coordinates t, x, y, z , instantaneous signalling between distant experimenters would not in itself be problematic. But what about the Lorentz transformation? One might be disturbed by the idea that an experimenter moving along (for example) the x -axis could ‘see’ such signals propagating ‘backwards in time’. However, a real experimenter does not simply ‘see’ the global time of his Lorentz frame. Rather, the experimenter has a collection of clocks distributed over space, which have to be *set* according to some chosen procedure. The time associated with an event occurring at some point in space is just the reading of the clock in the neighbourhood of that event. If an event B is for some physical reason regarded as ‘causing’ a spatially-distant event A (for example a message is sent from B to A), and if the reading of a clock at B is larger than the reading of a clock at A , then before declaring this paradoxical one ought to ask how the clocks at A and B were set in the first place.

If the moving experimenter chooses Einstein’s so-called ‘synchronisation’, using light pulses whose speed is taken to be isotropic, then at (preferred) time t the moving clock located at x, y, z will read a time

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}} \quad (13)$$

From our perspective, the interpretation of this formula is very simple. The moving clocks distributed along $x > 0$ have been initially set (for example at $t = 0$) to read progressively earlier times, with a lag proportional to x ; while the moving clocks along $x < 0$ have been similarly set to read later times. These settings have been chosen precisely so as to make a light pulse (with speed c in

the preferred frame¹⁶⁾ *appear* to have a speed c , along both $+x$ and $-x$, in the moving frame. This is the origin of the term $-vx/c^2$ (to lowest order in v/c). If one includes the effect of motion, which as we have said slows clocks down, one also obtains the factor $1/\sqrt{1-v^2/c^2}$.

If the moving experimenter adopts the Einstein convention for the synchronisation of clocks, then the settings (13) have the following peculiarity: an instantaneous signal propagating along $+x$ in the preferred frame appears to be going ‘backwards in time’ as judged by the moving clocks with settings t' . That is, if the signal starts at x_B and propagates to $x_A > x_B$, then if $v > 0$ the readings t'_A, t'_B of the moving clocks at the events A, B have the property $t'_B > t'_A$. But there is nothing mysterious or paradoxical here: for the moving clocks were initially set with a time-lag proportional to x , and the result $t'_B > t'_A$ is a direct and immediate reflection of this initial set-up. Indeed, this phenomenon is exactly the same as the familiar ‘jet lag’ which occurs when an experimenter moves rapidly from one time zone to another on the Earth’s surface. Clocks distributed over the Earth’s surface have been set according to a convention related to the locally-observed position of the Sun in the sky, and it is in no way surprising or problematic that a jet passenger may in a formal sense ‘travel backwards in time’.

Note that, from this point of view, time dilation is a real physical effect of motion which may be unambiguously verified by experiment (for example by taking one clock on an accelerated round trip and comparing it with an unaccelerated clock before and after). Whereas, the so-called relativity of simultaneity is merely the result of a convention about the way clocks are synchronised in different frames.

It is worth remarking that, as already mentioned in section 2.2, the origin of the term $-vx/c^2$ in (13) was clearly understood by Poincaré well before 1905. In a paper published in 1900 (Poincaré 1900), concerned mainly with action and reaction in electrodynamics, Poincaré (who works to lowest order in v/c) writes:¹⁷

‘I assume that observers situated at different points set their watches with the aid of light signals; that they try to correct these signals by the transmission time, but that ignoring their translatory motion and therefore believing that the signals are transmitted with equal speed in both directions, they content themselves with crossing the observations, sending a signal from A to B, then another from B to A. The local time t' is the time shown by watches set in this way.

If then $V = 1/\sqrt{K_0}$ is the speed of light, and v the speed of translation of the Earth which I assume parallel to the positive x -axis, one will have:

$$t' = t - \frac{vx}{V^2}$$

¹⁶The speed c in the preferred frame will of course be independent of the motion of the source, as expected of a wave phenomenon.

¹⁷For a detailed analysis of this paper by Poincaré, as well as for a reconstruction of Poincaré’s argument in the cited passage, see the paper by Darrigol (1995).

' (Poincaré 1900: 483; translation by the author).

Poincaré understood that moving experimenters who assume that the speed of light is still c in all directions would adjust their clocks at different points in space with settings that differ by the term $-vx/c^2$ (to lowest order in v/c).

If instead distant clocks are synchronised by nonlocal means, then the speed of light will be measured to be isotropic only in the preferred rest frame. In quantum equilibrium, of course, such nonlocal signalling is impossible and the true rest frame cannot be detected.

Note that, in the specific hidden-variables theory given by pilot-wave dynamics, even leaving nonlocality aside, the natural kinematics of the theory is arguably that of Aristotelian spacetime $E \times E^3$, with a preferred state of rest (Valentini 1997). This is essentially because the dynamics is first order in time, so that rest is the only reasonable definition of ‘natural’ or ‘unforced’ motion. Pilot-wave theory then has a remarkable internal logic: both the structure of the dynamics, and the operational possibility of nonlocal signalling out of equilibrium, independently point to the existence of a natural preferred state of rest.

4.2 Curved Spacetime

In the presence of gravitation, the above discussion may be extended to any classical background spacetime possessing at least one global time function t . This is hardly a restrictive requirement. For it is widely assumed that, classically, any physical spacetime must be globally hyperbolic¹⁸ – that is, must possess a Cauchy surface (a spacelike slice on which initial data determine the entire spacetime) – and it is a theorem that any globally hyperbolic spacetime has topology $\mathbb{R} \times \Sigma$ (where Σ is a Cauchy surface) (Hawking and Ellis 1973).

Consider, then, a curved spacetime that can be foliated (in general non-uniquely) by spacelike hypersurfaces Σ labelled by a global time function t . The classical spacetime metric may then be written in the form

$$d\tau^2 = {}^{(4)}g_{\mu\nu}dx^\mu dx^\nu = N^2 dt^2 - g_{ij}dx^i dx^j$$

where we have set the shift vector $N^i = 0$, so that lines $x^i = \text{const.}$ are normal to Σ . (This may always be done, as long as the lines $x^i = \text{const.}$ do not run into singularities.) The lapse function $N(x^i, t)$ measures the proper time lapse normal to Σ per unit of coordinate time t .

It may now be assumed that nonlocality acts instantaneously with respect to one of these foliations, denoted $\Sigma(t)$. There is then a true slicing, and spacetime is really the time evolution of the (absolute) 3-geometry $\mathcal{G}(t)$ of $\Sigma(t)$, with metric $g_{ij}(x^k, t)$ (Valentini 1992, 1996).

On this view, a small rod at time t has proper length

$$dl = (g_{ij}dx^i dx^j)^{1/2}$$

¹⁸See, for example, Penrose (1979).

while a clock at rest in 3-space ticks a proper time

$$d\tau = N(x^i, t)dt$$

If a clock moves a spatial distance dl in a time dt it will tick a proper time

$$d\tau^2 = N^2 dt^2 - dl^2$$

Some remarks are in order.

First, there is an asymmetry here between space and time. It is assumed that ordinary rods faithfully realise the true distance element dl of space. Whereas, we assume that ordinary clocks do not faithfully register true time t ; rather, their rate of ticking is affected by the local lapse field $N(x^i, t)$.

Second, note the difference from the case where gravity is absent. There we saw that moving clocks are slowed down. The same effect occurs here, but in addition, the rate of ticking of clocks is affected by their spatial location. There is a field $N(x^i, t)$ on 3-space which has a dynamical effect on the rate of clocks even when they are at rest.

Third, this interpretation does not necessarily involve the introduction of an independent field N on 3-space. For this field could be determined by the geometry of 3-space; N could, for example, be a simple fixed function of the 3-metric g_{ij} such as

$$N = g^{-1/2} \quad (g \equiv \det g_{ij})$$

(as in unimodular gravitation with $\det {}^{(4)}g_{\mu\nu} = 1$ (Unruh 1989)). Presumably, N will be merely an effective field, emerging from some more fundamental theory (possibly a quantum or subquantum theory of gravity). In this way, there could be an underlying dynamical origin for the phenomenological distortion of clock rates by the field N .

As in the flat case, one must be careful to distinguish between simultaneity and synchronicity. Clocks located at different spatial points x^i on the same hypersurface (with label t) record simultaneous events, but the field N causes even stationary clocks to tick at different rates and lose their synchrony. Thus, for example, let clocks at events A_1, B_1 at time t_1 (on the preferred spacelike hypersurface Σ_1) move along timelike lines to events A_2, B_2 at time $t_2 > t_1$ (on the preferred spacelike hypersurface Σ_2). Assume for simplicity that the clocks remain at rest in space. Then each clock will tick a proper time

$$\Delta\tau = \int_{t_1}^{t_2} N(x^i, t)dt$$

where the integral is taken along the respective path. The lapse function N will generally differ along the two paths A_1 – A_2, B_1 – B_2 . Thus, clocks synchronised at the simultaneous events A_1 and B_1 (using nonlocal signals) will no longer be synchronised at A_2 and B_2 , even though A_2 and B_2 are also simultaneous.

From a conventional perspective, this view will certainly seem eccentric, and indeed it would be in the absence of any evidence for nonlocality. But if one takes seriously Bell's deduction that nonlocal influences do occur in Nature,

and if one further accepts that our current inability to control these events is merely a contingency of a particular distribution of hidden variables, and bearing in mind that these effects occur at ordinary energies and macroscopic distances, then the above view provides a consistent phenomenological means of embedding such nonlocally-connected quantum events within the surrounding approximately classical spacetime.

Again, one need not view the above construction as fundamental. A microscopic theory of spacetime may well provide a very different picture at the fundamental level. But if one accepts the existence of nonlocality, then it seems natural that the above construction should emerge in some approximation.

In quantum equilibrium, of course, nonlocality and the true slicing cannot be detected, as in the case of flat spacetime. Possibly, the observed cosmological rest frame is a relic of early nonlocality – arising from quantum non-equilibrium in the early universe – and coincides with true rest (Valentini 1991b, 1992, 1996).

5 Discussion and Conclusion

We have presented a means of embedding quantum nonlocality within a background classical spacetime (flat or curved), by introducing an absolute simultaneity associated with a preferred foliation by spacelike hypersurfaces (where the preferred foliation defines a preferred local state of rest). It should be noted that this is unlikely to be the *only* way of constructing such an embedding. For as emphasised by Poincaré, the choice of geometry to be used in physics is really dictated by convenience. There is no question of proving that the most convenient choice is the only one possible, because one may always adopt a different geometry by adding appropriate compensating factors to the dynamics.

In his book *Science and Hypothesis* (Poincaré 1902), Poincaré illustrated this point in terms of an analogy with measuring rods affected by thermal expansion. Consider, for example, metal rods on a heated flat metal plate.¹⁹ If the temperature of the plate is non-uniform, and if all the rods have the same expansion coefficient, then (assuming the rods reach thermal equilibrium instantly) measurements within the surface using these rods will simulate the geometry of a curved 2-surface – that is, a non-Euclidean geometry. Creatures living on such a surface could believe it to be curved, as long as all their rods were affected by temperature in the same way. Equally, they could believe their surface to be flat, with all rods being universally distorted (expanded or contracted) by means of some agency acting upon them. There would be no way of telling the difference. However, the creatures may well come to think that, because the required distortions are the same for all rods, it is more convenient to ascribe the distortions to the geometry of space itself; that is, if the apparent geometry of the 2-surface is the same no matter which rods are used, then one may as well define the apparent geometry to be the actual geometry of space. As Poincaré put it:

¹⁹Poincaré’s example actually involved a 3-sphere within which the temperature varies as a certain function of radius.

‘Experiment tells us not what is the truest, but what is the most convenient geometry’ (Poincaré 1902).

The situation is no different in present-day physics. For example, instead of interpreting general relativity in terms of a curved spacetime with metric $g_{\mu\nu}$, it is possible to interpret it in terms of a Minkowski spacetime, with flat metric $\eta_{\mu\nu}$, containing a field $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ which distorts rods and clocks so as to give the appearance of curved spacetime (Weinberg 1972). It cannot be proved that spacetime is really curved; but, because the effects of the field $h_{\mu\nu}$ are universal – the same for all rods and clocks – it is more convenient to regard those effects as purely kinematical, that is, as part of the geometry with metric $g_{\mu\nu}$.

Similarly, classical special relativity may equally be interpreted in terms of a preferred (yet unobservable) rest frame, where motion with respect to the preferred frame has the dynamical effect of slowing clocks and contracting rods. As emphasised for example by Bell (1987), this is an equivalent formulation of the same physics. One may find it objectionable to have an underlying preferred frame which can never be detected (classically), but nevertheless this formulation of special-relativistic physics is consistent. It often happens that the same physics can be formulated in equivalent, empirically indistinguishable ways. Instead of insisting that non-standard formulations are ‘wrong’, it might be wiser to bear in mind that they might prove useful in some situations, and that in the future, as new physics is discovered, they might even turn out to be closer to the truth. The preferred-frame interpretation of special relativity certainly comes into its own in the face of quantum nonlocality.

These examples illustrate a general point. The division between kinematics and dynamics cannot be determined uniquely. There is a ‘shifty split’ between the two. Yet, it is convenient to define the kinematics (or spacetime geometry) so that it contains or summarises universal physical effects which are independent of (for example) the mass and composition of bodies. For this reason, universal symmetries such as Lorentz invariance are usually regarded as part of the kinematics, so that spacetime is defined as locally Minkowskian. However, with the discovery of new effects such as quantum nonlocality, the most convenient choice of spacetime geometry may have to be revised, as we have argued here.

From this ‘Poincaréan’ point of view, it seems misguided to try to argue that a certain kinematics – with or without an absolute simultaneity – *must* be adopted. One can only propose a certain kinematics and argue that it provides the simplest and most natural description of the phenomena.²⁰

We claim, then, that the above construction, with an absolute simultaneity (associated with a preferred foliation and a preferred local state of rest), is the natural one given the known facts; and, we suggest that it should emerge from

²⁰This is, in fact, arguably true not only regarding spacetime geometry, but also regarding physical laws in general, since it is always possible to write alternative formulations of the same physics.

a more fundamental theory in the limit of an approximately classical spacetime background.

Alternatively, one might try to develop a theory of nonlocal interactions on Minkowski spacetime. In itself, the mere fact of superluminal interaction is not necessarily incompatible with fundamental Lorentz invariance. For example, the interactions might be instantaneous in the centre-of-mass frame (a manifestly Lorentz-invariant statement). But then one must somehow make sense of backwards-in-time signals in other frames. This last question becomes particularly poignant if one is willing to consider quantum non-equilibrium and the associated practical signalling at a distance. Some workers, however, maintain that backwards-in-time effects should be allowed, arguing that these provide a loophole through which nonlocality may be avoided (Price 1996).²¹ Attempts have been made to formulate a de Broglie-Bohm-type theory of particle trajectories with fundamental Lorentz invariance, but it would appear that the dynamics (and the quantum equilibrium distribution) must be defined on a preferred spacelike slice, that is, in a preferred rest frame (Hardy 1992; Berndl and Goldstein 1994; Berndl *et al.* 1996). (See, however, Dewdney and Horton (2002) for an attempt to avoid this problem.) A similar result has been shown for any preferred local quantum observable (not necessarily particle positions) (Myrvold 2002). In evaluating the advantages and disadvantages of all these approaches, in our view, it ought to be remembered that spacetime structure is not a metaphysical *a priori* background onto which dynamics is to be grafted at all costs; rather, it is as subject to possible revision as dynamics itself.

It may well be that the issue of nonlocality vis à vis relativistic spacetime will only be settled upon making further progress in physics. From our perspective, for as long as we are confined to a state of statistical equilibrium that hides the underlying nonlocality from direct view, it seems probable that the argument will continue to be unresolved. On the other hand, if quantum non-equilibrium were to be discovered and used in practice for instantaneous signalling over remote distances, then in such circumstances it seems likely that physicists would see the convenience of adopting a global definition of absolute simultaneity.

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²¹The derivation of Bell's inequality assumes that the initial parameters λ are unaffected by the future settings of the equipment.

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